1.130
(a)

| p | $\mathrm{p} \oplus \mathrm{p}$ |
| :---: | :---: |
| T | F |
| F | F |

(b)

| p | $\neg \mathrm{P}$ | $\mathrm{p} \oplus \neg \mathrm{p}$ |
| :---: | :---: | :---: |
| T | F | T |
| F | T | T |

(c)

| p | q | $\neg \mathrm{q}$ | $\mathrm{p} \oplus \neg \mathrm{q}$ |
| :---: | :---: | :---: | :---: |
| T | T | F | T |
| T | F | T | F |
| F | T | F | F |
| F | F | T | T |

(d)

| p | q | $\neg \mathrm{p}$ | $\neg \mathrm{q}$ | $\neg \mathrm{p} \oplus \neg \mathrm{q}$ |
| :---: | :---: | :---: | :---: | :---: |
| T | T | F | F | F |
| T | F | F | T | T |
| F | T | T | F | T |
| F | F | T | T | F |

(e) (f)

| p | q | $\mathrm{p} \oplus \mathrm{q}$ | $\mathrm{p} \oplus \neg \mathrm{q}$ | $(\mathrm{p} \oplus \mathrm{q}) \vee(\mathrm{p} \oplus \neg \mathrm{q})$ | $(\mathrm{p} \oplus \mathrm{q}) \wedge(\mathrm{p} \oplus \neg \mathrm{q})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| T | T | F | T | T | F |
| T | F | T | F | T | F |
| F | T | T | F | T | F |
| F | F | F | T | T | F |

38
(a) $11000 \wedge(01011 \vee 11011)=11000 \wedge 11011=11000$
(b) $(01111 \wedge 10101) \vee 01000=00101 \vee 01000=01101$
(c) $(01010 \oplus 11011) \oplus 01000=10001 \oplus 01000=11001$
(d) $(11011 \vee 01010) \wedge(10001 \vee 11011)=11011 \wedge 11011=11011$
1.362
(a) $\forall x(P(x) \rightarrow \neg S(x))$
(b) $\forall x(R(x) \rightarrow S(x))$
(c) $\forall x(Q(x) \rightarrow P(x))$
(d) $\forall x(Q(x) \rightarrow \neg R(x))$
(e) Yes. If x is one of my poultry, then he is a duck (by part (c)), hence not willing to waltz (by part (a)). Since officers are always willing to waltz (by part (b)), $x$ is not an officer.

### 1.514

(a) Let $c(x)$ be " $x$ is in this class," let $r(x)$ be " $x$ owns a red convertible," and let $t(x)$ be " $x$ has gotten a speeding ticket." We are given premises $c$ (Linda), $r($ Linda ), $\forall x(r(x) \rightarrow t(x))$, and we want to conclude $\exists x(c(x) \wedge t(x))$.

| Step | Reason |
| :--- | :--- |
| 1. $\forall x(r(x) \rightarrow t(x))$ | Hypothesis |
| 2. $r$ (Linda) $\rightarrow t($ Linda $)$ | Universal instantiation using (1) |
| 3. $r$ (Linda) | Hypothesis |
| 4. $t$ (Linda) | Modus ponens using (2) and (3) |
| 5. $c$ (Linda) | Hypothesis |
| 6. $c$ (Linda) $\wedge t($ Linda $)$ | Conjunction using (4) and (5) |
| 7. $\exists x(c(x) \wedge t(x))$ | Existential generalization using (6) |

(b) Let $r(x)$ be " $r$ is one of the five roommates listed," let $d(x)$ be " $x$ has taken a course in discrete mathematics," and let $a(x)$ be " $x$ can take a course in algorithms." We are given premises $\forall x(r(x) \rightarrow d(x))$ and $\forall x(d(x) \rightarrow a(x))$, and we want to conclude $\forall x(r(x) \rightarrow a(x))$. In what follows $y$ represents an arbitrary person.

| Step | Reason |
| :--- | :--- |
| 1. $\forall x(r(x) \rightarrow d(x))$ | Hypothesis |
| 2. $r(y) \rightarrow d(y)$ | Universal instantiation using (1) |
| 3. $\forall x(d(x) \rightarrow a(x))$ | Hypothesis |
| 4. $d(y) \rightarrow a(y)$ | Universal instantiation using (3) |
| 5. $r(y) \rightarrow a(y)$ | Hypothesis |
| 6. $\forall x(r(x) \rightarrow a(x))$ | Conjunction using (4) and (5) |

(c) Let $s(x)$ be " $x$ is a movie produced by Sayles," let $c(x)$ be " $x$ is a movie about coal miners," and let $w(x)$ be "movie $x$ is wonderful." We are given premises $\forall x(s(x) \rightarrow w(x))$ and $\exists x(s(x) \wedge c(x))$, and we want to conclude $\exists x(c(x) \wedge w(x))$. In our proof, $y$ represents an unspecified particular movie.

| Step | Reason |
| :--- | :--- |
| 1. $\exists x(s(x) \wedge c(x))$ | Hypothesis |
| 2. $s(y) \wedge c(y)$ | Existential instantiation using (1) |
| 3. $s(y)$ | Simplification using (2) |
| 4. $\forall x(s(x) \rightarrow w(x))$ | Hypothesis |


| 5. $s(y) \rightarrow w(y)$ | Universal instantiation using (4) |
| :--- | :--- |
| $6 . w(y)$ | Modus ponens using (3) and (5) |
| 7. $c(y)$ | Simplification using (2) |
| 8. $w(y) \wedge c(y)$ | Conjunction using (6) and (7) |
| 9. $\exists x(c(x) \wedge w(x))$ | Existential generalization using (8) |

(d) Let $c(x)$ be " $x$ is in this class," let $f(x)$ be " $x$ has been to France," and let $l(x)$ be " $x$ has visited the Louvre." We are given premises $\exists x(c(x) \wedge f(x))$, $\forall x(f(x) \rightarrow l(x)$ ), and we want to conclude $\exists x(c(x) \wedge l(x))$. In our proof, $y$ represents an unspecified particular person.

| Step | Reason |
| :--- | :--- |
| 1. $\exists x(c(x) \wedge f(x))$ | Hypothesis |
| 2. $c(y) \wedge f(y)$ | Existential instantiation using (1) |
| 3. $f(y)$ | Simplification using (2) |
| 4. $c(y)$ | Simplification using (2) |
| 5. $\forall x(f(x) \rightarrow l(x))$ | Hypothesis |
| 6. $f(y) \rightarrow l(y)$ | Universal instantiation using (5) |
| 7. $l(y)$ | Modus ponens using (3) and (6) |
| 8. $c(y) \wedge l(y)$ | Conjunction using (4) and (7) |
| 9. $\exists x(c(x) \wedge l(x))$ | Existential generalization using (8) |

### 1.634

No. This line of reasoning shows that if $\sqrt{2 x^{2}+1}=x$, then we must have $x=1$ or $x=-1$. These are therefore the only possible solutions, but we have no guarantee that they are solutions, since not all of our steps were reversible (in particular, squaring both sides). Therefore we must substitute these values back into the original equation to determine whether they do indeed satisfy it.
1.736

|  | 8 | 5 | 3 |
| :--- | :--- | :--- | :--- |
| 1. | 8 | 0 | 0 |
| 2. $(8 \rightarrow 5)$ | 3 | 5 | 0 |
| 3. $(5 \rightarrow 3)$ | 3 | 2 | 3 |
| $4 .(3 \rightarrow 8)$ | 6 | 2 | 0 |
| $5 .(5 \rightarrow 3)$ | 6 | 0 | 2 |
| $6 .(8 \rightarrow 5)$ | 1 | 5 | 2 |
| $7 .(5 \rightarrow 3)$ | 1 | 4 | 3 |

